Problem Set 7 - Solution - LV 141.A55 QISS

1. Coulomb Blockade at Room Temperature

$$E_c = 300K \cdot k_B = 4.2 \cdot 10^{-24}\mathrm{J}$$

$$C = \frac{e^2}{2E_c} = 3.1 \cdot 10^{-18}\mathrm{F}$$

$$A = \frac{Cd}{\epsilon_0} = (19\mathrm{nm})^2$$

2. Single Electron Transistor

$$G(V_{\mathrm{SD}}, V_g, n_1, n_2) = \frac{((n_1 - n_2)e - C_g V_g)^2}{2C_\Sigma} - \frac{n_1 C_1 + n_2(C_2 + C_g)}{C_\Sigma} eV_{\mathrm{SD}}$$

(a)

$$\begin{aligned} \Delta G_{0\to 1} &= G(V_{\mathrm{SD}}, V_g, 1, 0) - G(V_{\mathrm{SD}}, V_g, 0, 0) \\ &= \frac{e}{C_\Sigma}\left(\frac{e}{2} - (C_g V_g + C_1 V_{\mathrm{SD}})\right) \end{aligned} \tag{1}$$

(b)

$$\begin{aligned} \Delta G_{1\to 0} &= G(V_{\mathrm{SD}}, V_g, 1, 1) - G(V_{\mathrm{SD}}, V_g, 1, 0) \\ &= \frac{e}{C_\Sigma}\left(-\frac{e}{2} + C_g V_g - (C_2 + C_g)V_{\mathrm{SD}}\right) \end{aligned} \tag{2}$$

(c) Tunneling through the first junction at zero temperature is allowed when the therm in the the bracket of equation (1) is negative. This condition is fulfilled above the line

$$V_{\mathrm{SD}} = \frac{1}{C_1}\left(\frac{e}{2} - C_g V_g\right)$$

This line crosses the V_g axes at $e/(2C_1)$ and has a negative slope of $-C_g/C_1$

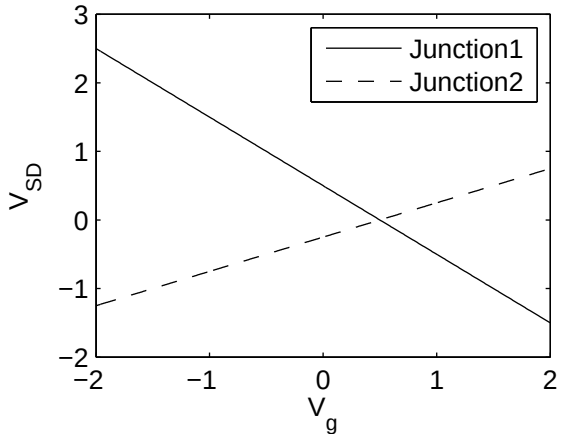

Figure 1: SET tunnel processes. Straight line: condition for the onset of tunneling through the first junction. Dashed line: condition for the onset of tunneling through the second junction.

Tunneling through the second junction at zero temperature is allowed when the therm in the the bracket of equation (2) is negative. This condition is fulfilled above the line

$$V_{\mathrm{SD}} = \frac{1}{C_2 + C_g}\left(C_g V_g - \frac{e}{2}\right)$$

This line also crosses the V_g axes at $e/(2C_1)$ and has a positive slope of $C_g/(C_2 + C_g)$
Current through the SET runs when both conditions are fulfilled, which is above both lines in figure (1).

(d) The reverse process $1 \leftarrow 0$, $0 \leftarrow 1$ is described by equations (1,2) but with a negative sign. Therefore current runs through the SET below the two lines in figure (2)

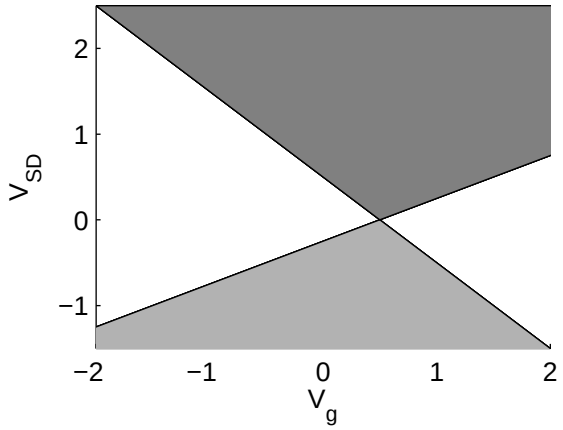

Figure 2: Current runs through the SET either above both lines or below both lines.

(e) Other processes are just shifted with respect of the V_g axes by integer multiples of C_g/e. In the white areas of figure (3) no tunneling is possible due to Coulomb blockade.

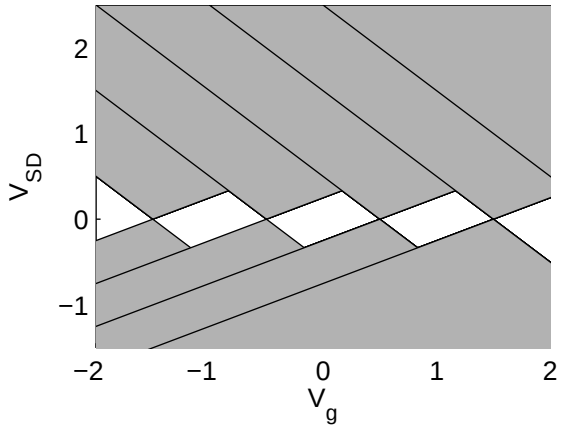

Figure 3: Allow possible tunnel processes.